Field Distribution of Two Conducting Posts in a Waveguide

STANISLAW L. LOPUCH AND T. KORYU ISHII, SENIOR MEMBER, IEEE

***Abstract*—This paper reports a field analysis of two short and narrow metal posts in a rectangular waveguide. The posts project from the center of the wider wall: one post is "grounded" in the wall, and the other post terminates in a variable impedance (coaxial line). An integral over a Green's function relates the post currents to the electric field, which is high and quite nonuniform near the posts, and depends on the variable impedance. The posts could excite a glow-discharge detector or mixer circuit. Some measurements of guide impedance and observations on a glow tube are included.**

I. Introducton

ONE OF THE basic problems in the design of waveguide components is the analysis of a post inserted into a waveguide. So far, most investigators were interested mainly in current distribution on a conducting post, or the impedance represented by such an obstacle [1]–[13]. To the authors' knowledge, little attention was paid to electric-field magnitude in the region of the obstacle.

Several publications mentioned above provided more or less explicit expressions for the calculation of electric-field disturbances caused by post-like obstacles, but no further analysis of field distribution was given. Such analysis seems to be especially useful if high-power devices are involved, or if the RF field distribution is important to the operation of the two-electrode devices studied extensively by Farhat and Kopeika *et al.* [14]–[17].

This paper reports an analysis of the electric-field distribution in the close vicinity of two small conducting posts in a waveguide. In the analyzed structure, one post is connected directly to the waveguide wall (grounded) and the other is externally loaded by a variable reactive impedance Z_L, as shown schematically in Fig. 1. The field distribution is analyzed as a function of Z_L, including mutual coupling between posts. To overcome some difficulties in the theoretical determination of post currents, a combination of a theoretical and experimental method is proposed to find the field strength in this structure.

II. Electric Field of a Single Post

To find a general expression for the scattered field due to the pair of posts, consider first a case where only one conducting post is inserted into the waveguide. Dimensions of the waveguide are such that only the TE_{10} mode can propagate. Distribution of the electric field, including all

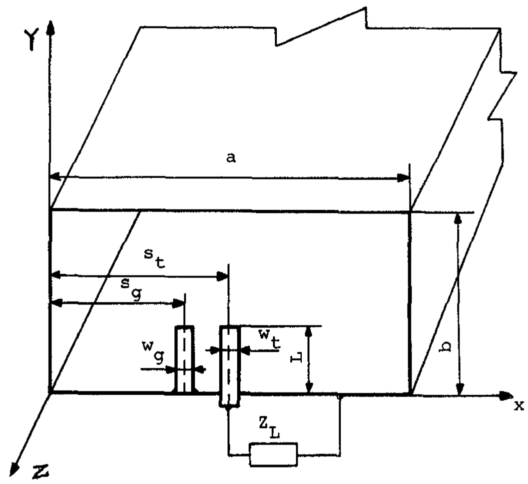


Fig. 1. Cross section of the rectangular waveguide showing a two-post structure under study. One post is grounded and the other is externally loaded.

possible evanescent modes radiated by the post, may be found in terms of a vector potential $\boldsymbol{A}$ associated with induced current on the post surface [12]

$$\boldsymbol{E}^s = -j\omega \boldsymbol{A} + \frac{\nabla(\nabla\cdot \boldsymbol{A})}{j\omega\epsilon\mu} \tag{1}$$

where the vector potentials are

$$\boldsymbol{A}(x,y,z) = \mu \iint_{s_0} \boldsymbol{G}(x,y,z|x',y',z')\boldsymbol{J}(y')\,da'. \tag{2}$$

The post current density $\boldsymbol{J}(y')$ has a y-component only, and hence the vector potential function $\boldsymbol{A} = \boldsymbol{a}_y A_y$. The appropriate Green's function that yields $\boldsymbol{A}$ from $\boldsymbol{J}$ may be taken as [2], [12]

$$\boldsymbol{G}(x,y,z|x',y',z') = \boldsymbol{a}_y\boldsymbol{a}_y \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \sin(k_x x)\sin(k_x x') \cdot\cos(k_y y)\cos(k_y y')\exp(-\Gamma_{mn}|z-z'|)$$

where

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b} \qquad k_0 = \frac{2\pi}{\lambda}$$

$$\Gamma_{mn} = \sqrt{k_x^2 + k_y^2 - k_0^2}$$

$$\delta_n = 1, \quad \text{for } n = 0$$
$$= 0, \quad \text{for } n \neq 0. \tag{3}$$

Manuscript received May 11, 1983; revised August 23, 1983.
S. L. Lopuch is with EPSCO, Inc., Westwood, MA 02090.
T. K. Ishii is with the Department of Electrical Engineering and Computer Science, Marquette University, Milwaukee, WI 53233.

0018-9480/84/0100-0029$01.00 ©1984 IEEE

For the purpose of this analysis, a rounded post of diameter d, small as compared to waveguide dimensions, may be represented by a flat strip of an equivalent width $w = 1.8d$ [2] placed in the plane $z = 0$. Alternative approaches are available [20]. But this approach was taken for its simpler mathematical treatment and complete dynamic field processes. In this model, the current distribution is assumed to be constant across the width of the strip. Current distribution along the length of the post or strip was a subject of controversy and some different models have been proposed [1]–[3], [13]; but with the simplifying assumptions already made, the most simple form, to satisfy the telegrapher's equations [12], [19], and boundary conditions at $y' = L$ and $y' = 0$

$$J(y') = \frac{I_0}{w}\sin\left[k_0(L - y')\right] \tag{4}$$

where k_0 is a free-space phase constant, seems to be sufficient. Placing the center of the strip at $x = s$, the vector potential was found to be [18]

$$A_y = \frac{\mu I_0}{w}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(2-\delta_n)}{ab\Gamma_{mn}}B_{mn} \cdot \sin(k_x x)\cos(k_y y)\exp(-\Gamma_{mn}|z|) \tag{5}$$

where

$$B_{mn} = \frac{k_0\left[\cos(k_0 L) - \cos(k_y L)\right]}{k_y^2 - k_0^2}\cdot\frac{2}{k_x}\sin(k_x s)\sin\left(k_x\frac{w}{2}\right).$$

Thus from (1), the scattered electric field close to the post may be represented as a function of space and current amplitude as

$$\begin{aligned} \boldsymbol{E}^s(x, y, z,) = j\frac{\eta I_0}{k_0^3 abw}\Bigg[&\boldsymbol{a}_x\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}C_{mn}B_{mn}k_x k_y \cdot\cos(k_x x)\sin(k_y y)\exp(-\Gamma_{mn}|z|)\right] \\ &+\boldsymbol{a}_y\Bigg[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}C_{mn}B_{mn}k_y^2\sin(k_x x)\cdot\cos(k_y y)\exp(-\Gamma_{mn}|z|) \\ &-\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}C_{mn}B_{mn}k_0^2\sin(k_x x)\cdot\cos(k_y y)\exp(-\Gamma_{mn}|z|)\Bigg] \\ &-\boldsymbol{a}_z\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}C_{mn}B_{mn}k_y\Gamma_{mn}\sin(k_x x)\cdot\sin(k_y y)\exp(-\Gamma_{mn}|z|)\right]\Bigg] \end{aligned} \tag{6}$$

where

$$C_{mn} = \frac{(2-\delta_n)(k_0^2 - k_y^2)}{\Gamma_{mn}}.$$

In (6), the electric-field components other than E_y^s are generated due to the term $\nabla(\nabla\cdot\boldsymbol{A})$ in (1). The current amplitude I_0 can be found in terms of the incident field amplitude E_0 by considering the total tangential component of the electric field at the surface of the electrode. For a perfect conductor

$$E_0\sin\frac{\pi x}{a} + E_y^s = 0, \qquad \text{at } s - \frac{w}{2} < x < s + \frac{w}{2}, \quad 0 < y < L \quad z = 0. \tag{7}$$

In practice, I_0 may be calculated from a measurement of the reflection coefficient in the waveguide. Since waveguide dimensions were assumed to be such that only the TE_{10} mode can propagate and all other higher modes are evanescent, at a reasonable distance from the post $z = z_1$, the scattered field corresponds to the first term of the E_y^s component in (6) and represents a reflected wave

$$\begin{aligned} \boldsymbol{E}^s(z_1) &= -\boldsymbol{a}_y j\frac{\eta I_0}{kabw}\cdot C_{10}B_{10}\sin\frac{\pi x}{a}\exp(-\Gamma_{10}|z_1|) \\ &= \boldsymbol{a}_y E_{y_{10}}^s(z_1). \end{aligned} \tag{8}$$

Taking the reflection coefficient at $z = z_1$

$$R(z_1) = \frac{E_{y_{10}}^s(z_1)}{E_{\text{in}}(z_1)}. \tag{9}$$

The current amplitude is

$$I_0 \simeq E_0\frac{jkabw}{\eta C_{10}B_{10}}R(z_1). \tag{10}$$

With I_0 known, the total field distribution normalized to the amplitude of the incident wave in the vicinity of the post can be found easily using (6).

III. Two Posts

In the system of two conducting posts, shown in Fig. 1, the field radiated by one post affects the current on the other, and vice versa. The scattered fields are generally not in phase with the incident field. The scattering from one post results in a change in the effective incident on the other, and thus the current amplitude is also changed.

Using a condition that a tangential field component at the surface of each post is zero, it would be possible to set up a system of two equations similar to (7), including in each case the field scattered from the other post, and then calculate both current amplitudes.

However, a more practical method is proposed. Each post related separately introduces a certain impedance to the waveguide. The mutual influence of two posts can be represented as an additional mutual coupling impedance Z_M. Denoting Z_g as the impedance of grounded post and Z_T as the total impedance represented by the terminated post, an equivalent circuit of such a structure including mutual impedance is shown in Fig. 2. It corresponds to one proposed by Chang and Khan [6].

In the experimental structure, the external load was realized as a coaxial line, thus Z_T can be represented further by an equivalent circuit given by Lewin [1]. Assuming, for simplicity, that the terminated post is placed at the center of the wider wall so that the transformation ratio

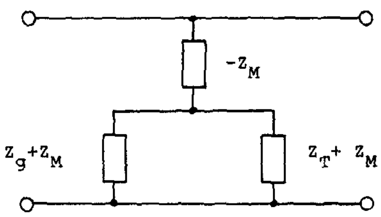


Fig. 2. General equivalent circuit for the pair of coupled posts.

corresponding to the position of coaxial entry becomes 1:1, a total equivalent circuit of the two-post obstacle is shown in Fig. 3. In this circuit, $Z_t = Z_g$ represents the impedance of the terminated post itself, Z_L is the input impedance of the coaxial line, and Z_A is added to reflect a change of the coaxial input impedance between the plane where Z_L was measured and the plane where it was effectively connected to the post.

Impedances Z_g, Z_H, Z_A, and Z_M can be found by measuring an impedance Z seen from the waveguide. A procedure to determine these impedances is as follows.

1) Inserting only the grounded post into the waveguide, one measures an additional impedance Z introduced in the waveguide, giving $Z_g = Z$.

2) Inserting only the terminated post, one measures the impedance seen from the waveguide when Z_L is short and open; this allows one to calculate Z_H and Z_A.

3) Inserting both electrodes, one measures the impedance seen from the waveguide with Z_L short and open; this allows one to calculate Z_M, provided that all other impedances are already known.

Making use of the equivalent circuit shown in Fig. 3, it is easy to show that the post current amplitudes ratio, including mutual coupling, is

$$\frac{I_g}{I_t} = 1 + \frac{Z_B}{Z_g + Z_M} \tag{11}$$

where I_g and I_t are currents on the grounded and terminated posts, respectively, and

$$\frac{1}{Z_B} = \frac{1}{Z_H} + \frac{1}{Z_A + Z_L}.$$

The total impedance seen from the waveguide is

$$Z = -Z_M + \frac{(Z_g + Z_M)(Z_g + Z_M + Z_B)}{2Z_g + 2Z_M + Z_B}. \tag{12}$$

The total current

$$I = I_g + I_t \tag{13}$$

may be measured using the method outlined in Section II, and thus both current amplitudes are determined by (11) and (13).

The scattered field from each post is described by (6), which can be represented as

$$E_g^s(x, y, z) = I_g F(z, y, z, B_{mn}(s_g, w_g))$$
$$E_t^s(x, y, z) = I_t F(z, y, z, B_{mn}(s_t, w_t)) \tag{14}$$

respectively, for grounded and terminated posts, where s_g and s_t are the x-coordinates of the post centers, w_g and w_t are the widths of the grounded and terminated posts, and $\boldsymbol{E}$ is a vector function. An analysis of (11) allows one to distinguish two extreme cases.

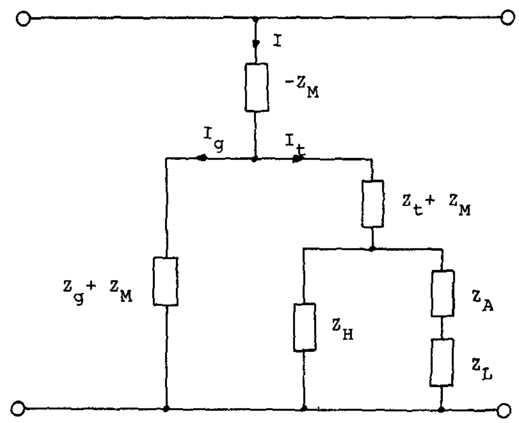


Fig. 3. Total equivalent circuit for the structure under analysis.

Case 1: When $Z_B = -(Z_g + Z_M)$, the current amplitude ratio is

$$\frac{I_g}{I_t} = 0.$$

If the total current I is not zero under this condition, the grounded post current must be equal to zero, and $I_t = I$.

Case 2: When $Z_B = \infty$, $I_t = 0$ and $I_g = I$.

Consequently, the total electric field in these two extreme cases is

$$\boldsymbol{E}(x, y, z) = \boldsymbol{E}_{in} + I\boldsymbol{F}(x, y, z, B_{mn}(s_t, w_t)) \tag{15}$$

when

$$Z_B = -(Z_g + Z_M)$$

and

$$\boldsymbol{E}(x, y, z) = \boldsymbol{E}_{in} + I\boldsymbol{F}(x, y, z, B_{mn}(s_g, w_g))$$

when

$$Z_B = \infty. \tag{16}$$

Since Z_B is a function of Z_L, this clearly indicates that the total electric field is different at each post, and the distribution of the field depends on the external microwave impedance connected to the system.

IV. Example

The total electric-field distribution was calculated for a particular structure. Two conducting posts of length $L = 1.0$ cm and diameter 0.9 mm were placed inside an S-band waveguide ($a = 0.214$ cm, $b = 3.409$ cm) symmetrically around the center of the wider waveguide wall with a separation distance $d = 2$ mm. The operating frequency was 2.45 GHz. Impedances of the post structure and adaptor were measured as outlined in Section III. The obtained values (normalized to the impedance of the waveguide) were

$$z_g = j\,3.33,\ z_H = -j\,1.43,\ z_A = j\,0.31,\ \text{and}\ z_m = j\,1.87. \tag{17}$$

The absolute value of relative total electric-field magnitude

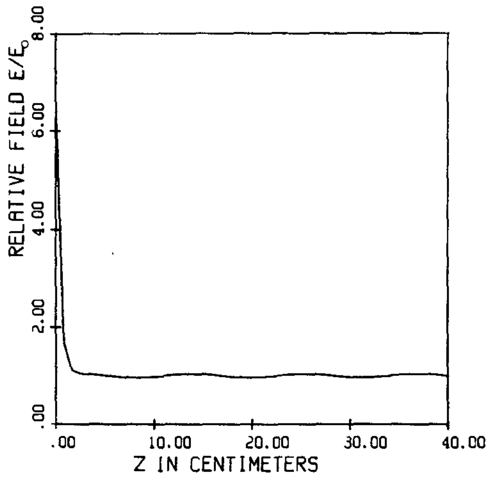


Fig. 4. Relative total electric field along the z-axis. Current on the grounded post compensated.

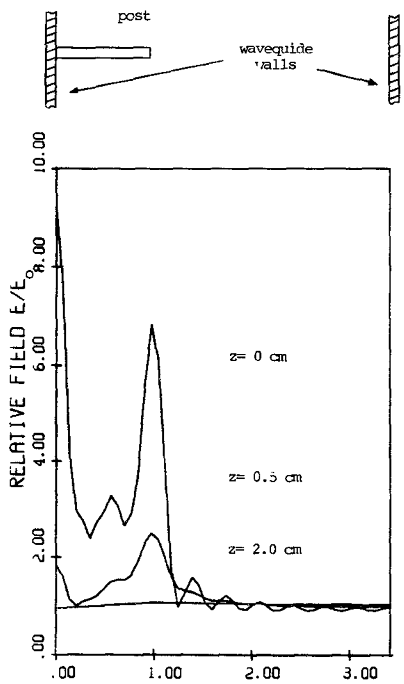


Fig. 5. Relative total electric field along the y-axis. Current on the grounded post compensated. Position of the posts is sketched in the upper part.

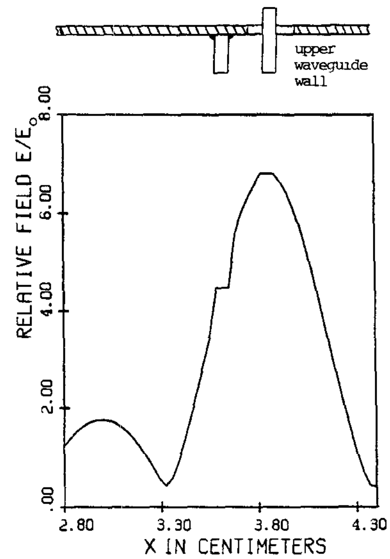


Fig. 6. Relative total electric field along the x-axis near the posts. Current on the grounded post compensated.

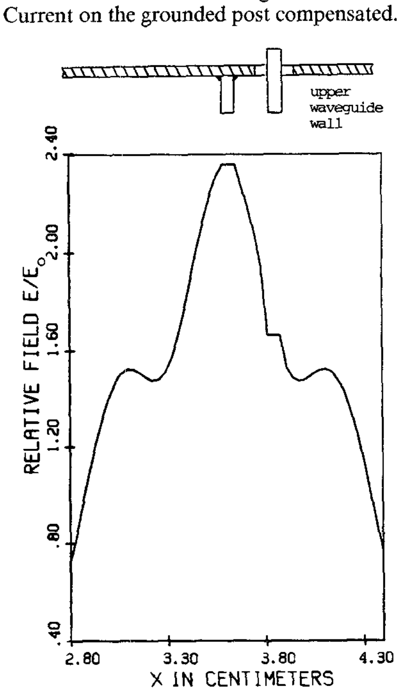


Fig. 7. Relative total electric field along the x-axis near the posts. Current on the terminated post compensated.

was calculated as

$$\left|\frac{E}{E_0}\right| = \left|\frac{E_{\text{in}} + E_t^s + E_g^s}{E_0}\right| \tag{18}$$

where E_g^s and E_t^s are defined by (14) and (6). The first 20 terms of infinite summation appearing in an expression for E^s were taken into account and results were calculated and plotted by a computer for a given load impedance Z_L, which defines the posts' currents.

An example of field distribution along the z-axis of the waveguide (toward the generator) at $x = 3.857$ cm (center of the terminated post and $y = 1.0$ cm, lower tip of the post) is shown in Fig. 4 for a case when $Z_B = -(Z_g + Z_M)$ i.e., $I_g = 0$. The main disturbance caused by the posts placed at $z = 0$ extends in the $\pm z$-directions to a distance of about 3 cm. Farther away from the posts, the field has a standing-wave pattern with a period of $\lambda_g/2$ and SWR of about 1.3. The measured VSWR under the same load conditions was 1.37.

Note that the field intensity close to the post is about seven times higher than E_0. The increase of the field can be represented by higher order modes radiated by the post, which cannot propagate along the waveguide, and thus their energy is stored in the region.

The field distribution along the y-axis for the same case is shown in Fig. 5. At a distance of 2 cm away from the post, the field distribution is uniform, as it is in the TE_{10} mode. It is interesting to note that on the post or at 0.5 cm away from the post, the field distribution is not uniform along the short post. There are bumps at $y = 0.5$ cm. These nonuniform distributions are attributed to the spatial harmonics as seen in (14). Figs. 6 and 7 show field distributions along the x-axis in the region very close to the posts for two extreme cases, $I_g = 0$ (i.e., $Z_B = -(Z_g + Z_M)$) and $I_t = 0$ (i.e., $Z_B = \pm j\infty$). It is apparent that the load impedance does not only change the field gradient and position of field maximum, but also affects strongly the magnitude of the total field.

The field distribution variation near the post was checked experimentally by observing ignition of a small neon-filled lamp inserted into a waveguide. In this experiment, the lamp's wire electrodes represent the posts analyzed above. A lead wire of one of the post-electrodes is directly grounded to the waveguide. The electrode represents a grounded post. The other post-electrode is connected to a variable external load impedance. This electrode represents the terminated post. Gas ignition pattern due to RF field only was observed as a function of external load impedance Z_L. It is claimed that the most intensive discharge corresponds to a maximum of local electric field. The ignition brightness level distribution along the posts closely corresponded to the calculated relative field strength and the predicted position of the maximum field, thus qualitatively confirming the present analysis.

IV. Conclusion

The foregoing sections illustrate a method of analysis of electric-field distribution near a system of two posts. Although the analysis presented was somewhat simplified by certain assumptions, and numerical values must be interpreted with care, the results obtained showed good correlation to experimental observation.

The analysis revealed that a not-so-obvious field variation along even a short post does exist. Also, the field distribution (magnitude and gradient) is strongly affected by the external load.

The above findings explain some observations encountered in discussions on the mechanism of glow discharge microwave detection [15]. The RF field adds to the dc field of a gas discharge structure, affecting detection sensitivity depending on the external load, which was also experimentally verified by the authors, but was not included in previous studies [15].

References

[1] L. Lewin, "A contribution to the theory of probes in waveguides," Inst. Elec. Eng., Mono. 259R, Oct. 1957.

[2] R. L. Eisenhard and P. J. Khan, "Theoretical and experimental analysis of a waveguide mounting structure," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 706–719, Aug. 1971.

[3] J. A. Bradshaw, "Scattering from a round metal post and gap," *IEEE Trans. Microwave Theory Tech.* vol. MTT-21, pp. 313–322, May, 1973.

[4] K. Chang and P. Khan, "Analysis of a narrow capacitive strip in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 536–541, May, 1974.

[5] O. L. El-Sayed, "Impedance characterization of two-post mounting structure for varactor-tuned Gunn oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 769–776, Aug. 1974.

[6] K. Chang and P. J. Khan, "Analysis of two narrow transverse resonant strips in waveguide," *Proc. IEEE*, pp. 1619–1620, Nov. 1976.

[7] J. S. Joshi and J. A. F. Cornick, "Analysis of a waveguide mounting configuration for electronically tuned transferred-electron-device oscillators and its circuit application," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 573–584, Sept. 1976.

[8] K. Chang and P. J. Khan, "Coupling between narrow transverse inductive strips in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 101–105, Feb. 1976.

[9] R. L. Eisenhart, "Discussion of a 2-gap waveguide mount," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 987–989, Dec. 1976.

[10] O. L. El-Sayed, "Generalized analysis of parallel two-post mounting structures in waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 24–33, Jan. 1977.

[11] J. S. Joshi and J. A. F. Cornick, "Analysis of waveguide post configurations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 169–181, Mar. 1977.

[12] R. E. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960.

[13] T. K. Ishii, *Microwave Engineering.* New York: Ronald Press, 1966.

[14] N. H. Farhat, "Optimization of millimeters wave glow discharge detectors," *Proc. IEEE*, vol. 62, pp. 279–280, Feb. 1974.

[15] N. C. Kopeika, "On the mechanism of glow discharge detection of microwave and millimeter wave radiation," *Proc. IEEE*, vol. 63, pp. 981–982, June 1975.

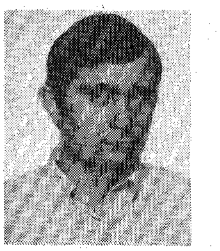
[16] N. S. Kopeika *et al.*, "Commercial glow discharge tubes as detectors of X-band radiation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 843–846, Oct. 1975.

[17] ——, "IF conversion gain of glow discharge lamps as X-band mixers for high LO power levels," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 227–232, Mar. 1979.

[18] S. L. Lopuch, "An investigation on direct current extraction from microwave induced plasma," Ph.D. dissertation, Marquette University, Milwaukee, WI, Dec. 1981.

[19] E. L. Ginzton, *Microwave Measurement.* New York: McGraw-Hill, 1957.

[20] P. Morse and H. Feshbach, *Methods of Theoretical Physics.* New York: McGraw-Hill, 1953.



Stanislaw L. Lopuch was born in Wroclaw, Poland, on November 20, 1953. He received the M.S. degree in electronics engineering from the Technical University of Wroclaw, Poland, in 1977, and the Ph.D. degree in electrical engineering from Marquette University, Milwaukee, WI in 1982.

In 1982, he joined the faculty of the Department of Electrical Engineering and Computer Science of Marquette University. He is currently with EPSCO Microwave, Inc.

T. Koryu Ishii (M'55–SM'65) was born in Tokyo, Japan, on March 18, 1927. He received the B.S. degree in electrical engineering from Nihon University, Tokyo, in 1950, M.S. and Ph.D. degrees in 1957 and 1959, respectively, in electrical engineering from the University of Wisconsin, Madison. He received the Ph.D. degree in engineering from Nihon University in 1961.

From 1949 to 1956, he worked on the research of microwave circuits and amplifiers and instructed students at Nihon University. From 1956 to 1959, he worked, particularly, on the research of the noise figures of microwave amplifiers at the University of Wisconsin. Since 1959, he has been with Marquette University, Milwaukee, WI, where at present he is a Professor of Electrical Engineering. The research areas include millimeter waves and microwave ferrite devices, thermionic and solid-state devices, circuit components and transmission lines, applications of microwaves, millimeter waves, and quantum electronics.

Dr. Ishii is a member of Sigma Xi, Eta Kappa Nu, Sigma Phi Delta, Tau Beta Pi, ASEE, AAUP, PCM, WSPE, NSPE, IECE of Japan, and is a registered Professional Engineer in the State of Wisconsin.